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Validation of Daganzo's Behavioral Theory of Mult-Lane Traffic Flow: Interim Report, Work Accomplished at SDSU

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INTRODUCTION

This report documents results of work completed at San Diego State University during the first year of a research project intended to validate a behavioral traffic-flow theory recently proposed by Daganzo (1,2). Work on this project completed at the University of California at Berkeley will be documented in a separate interim report.

In the idealized form developed so far, Daganzo's theory assumes two types of drivers, aggressive (*rabbits*) and timid (*slugs*), and two lane groups, shoulder lanes and passing lanes. In free flow, rabbits travel faster than slugs and the two groups are segregated in what Daganzo terms *two-pipe* flow, with the slugs all in the shoulder lane and the rabbits all in the passing lane. In high-volume uncongested flow, rabbits will follow one another with very small headways, so long as they are able to pass, and such drivers are referred to as being *motivated* because the very small headways are held to be motivated by the desire to pass slower vehicles. Whenever an event occurs that reduces speed in the passing lane to (or below) v_f , the free-flow speed in the shoulder lane, the rabbits change lanes to equalize speeds, lose their "motivation," and increase their headways. This results in *one-pipe* flow. Because of the difference in the headways, maximum uncongested flow rates in the passing lane exceed the queue discharge rate for this lane. Daganzo provides solutions for a number of cases involving transitions from two-pipe to one-pipe flow and vice versa. These solutions are presented for the most part in terms of wave speeds and traffic states in, upstream, and downstream of queues.

The major objective of the research project is validation of Daganzo's theory. This includes a thorough review of past empirical work related to the theory and verification of a set of predictions, including those proposed by Daganzo and others that can be derived from the theory. Predictions stated or clearly implied in references 1 and 2 include the following:

1. *A traffic "collapse" from an oversaturated state on the passing lane is possible even if the shoulder lane flow is low.*
2. *A fast wave and a precursor state occur behind the back of a 1-pipe queue only if the approaching flows are high.*
3. *A "reverse collapse" is possible, particularly in 2-lane freeways: i.e. if the proportion of slugs is so low that (capacity) flow in a 1-pipe queue is higher than the highest possible uncongested flows, then by restricting the speed of an uncongested traffic stream with maximal flows to a value slightly below v_f , one could force it into a one-pipe regime and increase total flow. [A more straightforward proposition is that*

if shoulder lane flow in the uncongested state is low enough, flow across all lanes will increase after a transition from uncongested to congested flow.]

- 4. Observers downstream of an incident that has just been removed will first see fast cars, then a 2-pipe “discharge state” with a consistent flow on the passing lane(s) (probably on the order of 2000+ veh/hr), and that if the flow on the shoulder lane is low (so that total flow in the discharge state is less than capacity) this will be followed by a capacity state with higher flows in the shoulder lane and slightly lower speeds on the passing lane.*
- 5. A 2-pipe discharge state may be seen upstream of an incident (after it is removed) after the capacity state if the stream is rich in slugs.*
- 6. An observer a considerable distance downstream of a merge bottleneck will see first a drop in flow in the passing lanes but with speed in the passing lanes greater than in the shoulder lane, then a reduction of flow in the passing lane, a more even distribution of flow across the lanes, and an increase in overall flow.*
- 7. Immediately downstream of the entrance ramp at a merge bottleneck an observer will see first an increase in flow across all lanes, then a reduction in speed in the passing lane, then an increase in shoulder lane flow accompanied by a gradual decrease in speed and flow in the passing lane, then a sharp reduction in speed across all lanes, and finally the capacity state (in which flow is less than in the “motivated” state prior to the collapse).*

In addition to these hypotheses, several others can be developed as extensions of the basic theory. So far, two additional hypotheses have been developed. These are explained in the following section.

In addition to development of additional hypotheses, project activities to date have included the review of past research, selection of a number of case study sites and data sets, and completion of two case studies based on data from the San Diego ramp metering system.

EXTENSIONS

To date, two additional hypotheses have been derived. The first of these has to do with the concept of motivation. Daganzo states that when speeds in the passing lane decrease below the free-flow speed in the slow lane, rabbits lose their motivation to maintain small headways because they are no longer able to pass other vehicles. This implies that, in some sense, the increase in headways in the passing lane that occurs at flow breakdown is the result of voluntary action on the part of the rabbits. The headways themselves may be thought of as consisting of two parts, the passage time of the lead vehicle, and the time gap between the rear of the lead vehicle and the front of the trailing vehicle. That is,

$$h = L/u + g \tag{1}$$

where h = headway
 L = length of the lead vehicle
 u = speed
 g = time gap

From equation 1, it is obvious that an increase in headways could be the result of the increase in passage times which will always occur when traffic slows down, or from an increase in time gaps. In order for the concept of motivation to be meaningful, it would seem that the increase in headways ought to involve an increase in the rabbits' time gaps (the voluntary component of the headway) as well as an increase in passage time. Accordingly, the following additional hypothesis is proposed:

8. *Average time gaps in the passing lane will increase when speeds in the passing lane drop to the level of those in other lanes.*

Average time gaps may be calculated from commonly-available flow and occupancy data as follows

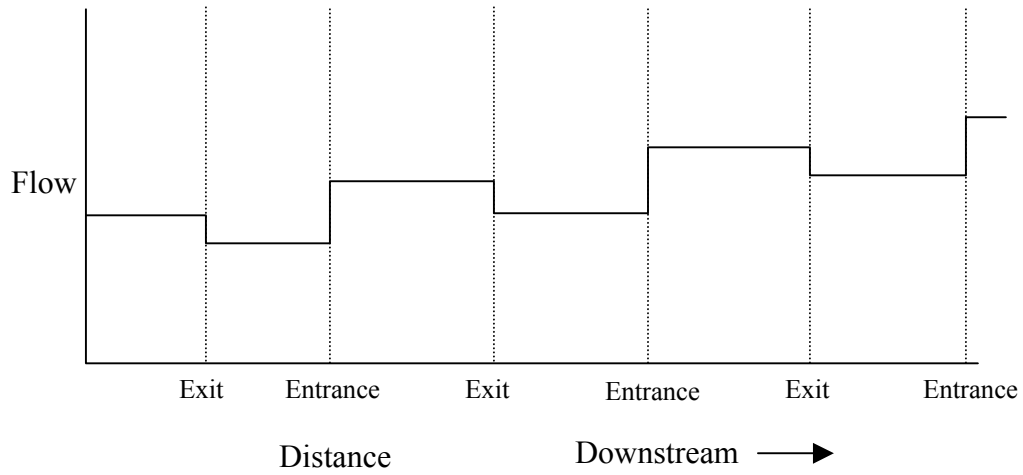
$$\bar{g} = \frac{1 - \Omega}{q} \quad (2)$$

where \bar{g} = average time gap
 Ω = occupancy, expressed as a fraction
 q = flow in vehicles per unit time

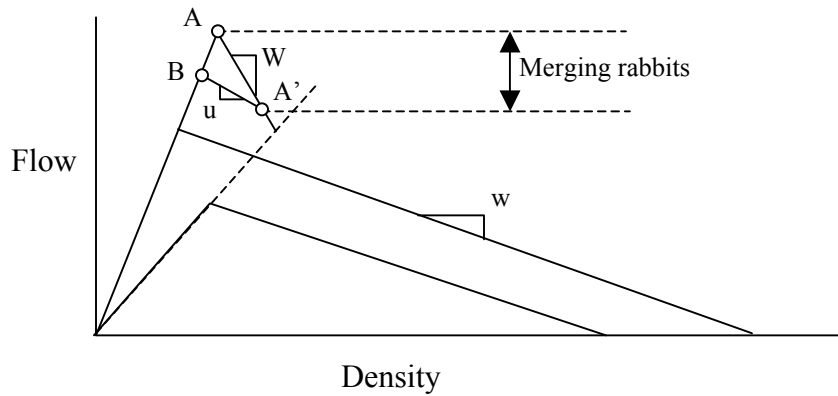
Note that the average gap will be in the same time units as the flow, so that calculation of gaps in seconds from flows in veh/h requires a unit conversion.

A second extension may be required to adapt the theory to metered freeway systems. Daganzo outlines a sequence of events triggering flow breakdown at a merge bottleneck where ramp volumes are increasing with time. In this sequence, a semi-congested state may form in the passing lane prior to the traffic collapse. If this occurs, flow in the passing lane is controlled from downstream and can no longer increase; consequently, collapse can come only from an increasing volume of rabbits from the entrance ramp merging into the passing lane. This fails to explain how traffic collapses can occur on metered freeways if the meter has already stabilized the ramp flow (often at the minimum metering rate) before the collapse occurs, but the freeway flow continues to increase. A hypothesis has been developed for the case in which there is a flow pattern that increases in the downstream direction, and flow at the merge point for the passing lane downstream of the critical ramp eventually reaches a level that puts it into a semi-congested state.

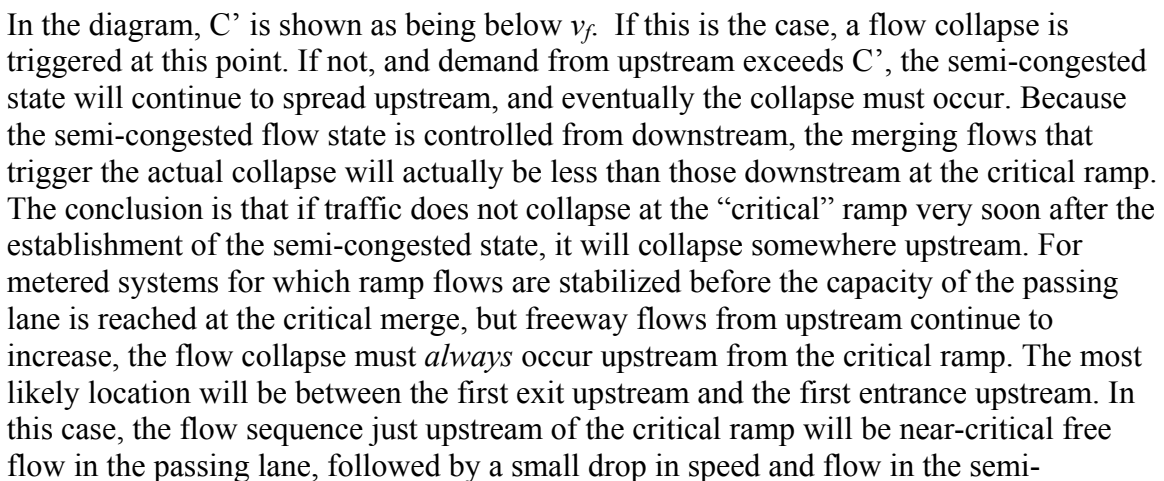
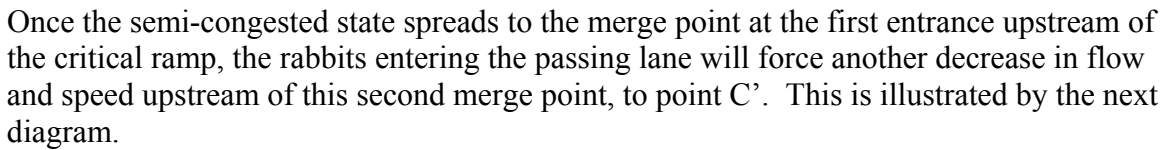
Assuming alternating on- and off-ramps, the flow profile (at an instant of time) for such a freeway is as follows:



The flow diagram just upstream of the point at which the rabbits from the critical ramp merge into the passing lane is as follows:



In the diagram, point B represents the rabbit demand flow arriving from upstream, and point A' represents the semi-congested flow that spreads upstream once flow B plus the merging rabbits exceeds Q_c , the critical flow in the passing lane. Since the semi-congested flow is controlled by conditions at its downstream end, flow upstream of this point can increase only if rabbits exit the lane. This will happen at some point upstream of each exit. The figure below shows the potential increase in flow to point B', which is equal to flow A' plus the exiting rabbits. If the demand upstream of the exit point is less than B', the semi-congested state will stabilize at this point. If not, it will continue to spread upstream. In the most likely case, demand from upstream will not exceed B' initially, but as demand from upstream continues to increase, it eventually will.



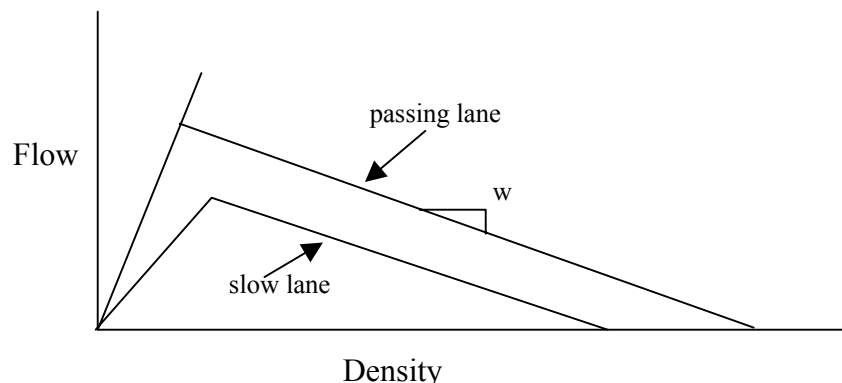
congested state, followed by discharge flow from the collapse upstream (with speed $< v_f$, but flow in the passing lane = Q_D).

For unmeted freeways where ramp flows continue to increase, there could be two collapses: one at the bottleneck resulting from the increasing rabbit flow from the ramp merging into the passing lane just downstream from the critical ramp, and another upstream, as merging rabbit flow from the next ramp upstream forces the semi-congested flow below v_f . Since the wave at the upstream end of the semi-congested state is a fast wave, semi-congested flow will normally exist for some time before at the upstream ramp before the congested flow reaches this point. This provides a window of time in which the second collapse can occur.

Based on these considerations, an additional hypothesis is proposed, as follows:

9. *If meters hold the on-ramp flows steady before the time of flow breakdown, the actual flow collapse will take place upstream of the critical ramp. Semi-congested states (involving a small drop in flow and speed in the left lane, but speeds in the left lane higher than those in the right lane) will be observed at detector locations upstream of the critical ramp prior to the flow collapse.*

In addition to these two hypotheses derived from the theory, consideration must be given to ways to adapt it to freeways that are wider than two lanes in each direction. Most of the assumed differences in behavior between rabbits and slugs may be related to the shape of flow-concentration relationships. In the original theory, the flow-concentration relationship for the passing lane is assumed to have a reversed-lambda shape in which the uncongested portion of the relationship extends above the maximum congested flow rate for the lane. The “stub” of the lambda represents the rabbits in their “motivated” state. The relationship for the slow lane, on the other hand, is assumed to be shaped like an inverted V. The uncongested portion of the relationship for the slow lane has a smaller slope that that of the passing lane, corresponding to the lower speed in this lane. The congested portions of the relationships are assumed to have identical slopes (corresponding to identical wave speeds), but that of the slow lane is displaced downward relative to that of the passing lane – that is, for a given density or occupancy, flows in the slow lane are less than those in the passing lane. The figure below illustrates the overall relationship as assumed by Daganzo.



It should be emphasized that these assumed flow-concentration relationships are idealizations and may not always be observable in practice. For instance, the reversed-lambda shape in the passing lane will be observed only if there are enough rabbits to fill the lane. Failure to observe a reversed-lambda flow-concentration relationship in the passing lane immediately downstream of a bottleneck does not necessarily invalidate Daganzo's theory in general, but it does mean that it does not apply to this location – specifically, that loss of motivation by rabbits is not the cause of flow breakdown.

Several possibilities exist for traffic in other lanes. One possibility is that several lanes will act as either passing lanes or slow lanes, and that all lanes in each category will display flow-concentration relationships similar to those of the extreme cases presented in Daganzo. A second possibility is that there will be intermediate classes of drivers in the intermediate lanes, so that these lanes will have distinctive patterns of their own. In order for the underlying logic of the theory to be valid, the inner (leftmost) and outer (rightmost) lanes must display the flow-concentration patterns assumed in the theory; patterns in intermediate lanes may resemble one or the other of the extremes, or fall somewhere in between them. Because the theory assumes identical wave speeds for all lanes in congested flow, however, congested-flow relationships for all lanes should have the same slope. These considerations lead to the following hypotheses about flow-concentration relationships:

10. *Flow-concentration plots for the left lane at locations downstream from bottlenecks will display a reversed-lambda shape.*
11. *Flow-concentration plots for the right lane will display an inverted-V shape; the central tendency of the congested-regime data will be parallel to that of the left lane, and will be displaced downward (i.e., lower flow for the same occupancy for the entire congested flow range).*
12. *Flow-concentration plots for intermediate lanes may be either reversed-lambda or inverted-V. Their shapes and displacements will indicate whether they are to be considered passing lanes, slow lanes, or some intermediate type with hypothetical driver types of their own.*

REVIEW OF EMPIRICAL TRAFFIC FLOW RESEARCH

Daganzo's theory may best be thought of as a limited generalization of the kinematic wave theory of Lighthill and Withem (3). As such, it incorporates the phenomena commonly described by kinematic wave theory, such as the movement of shock waves, and seeks to explain other interesting phenomena that have been observed, especially the decrease in flow in the passing lane that is often observed downstream of bottlenecks when flow breaks down. Daganzo's theory is based on a review of previously-known empirical evidence that is summarized in (1). The resulting theory is claimed to be "qualitatively consistent with all the empirical observations (old and new) known to this author." A literature review was undertaken to verify the extent to which the theory really is consistent with past empirical findings. This review addressed two primary issues:

1. The extent to which the facts forming the principal bases of the theory are established by previous empirical research.
2. The scope of the theory relative to the overall body of empirical knowledge related to traffic flow – that is, the extent to which it addresses the full range of phenomena described in the empirical literature.

The principal bases of the theory include the following assumptions, which are believed to be supported by past empirical research:

1. Traffic in uncongested flow is segregated by lane in terms of speed, with the highest speeds typically found in the median lane. This speed segregation breaks down in the transition to congested flow.
2. The maximum flow rate in the passing lane (typically the lane nearest the freeway median) occurs in prior to the transition to congested flow and is greater than the maximum flow rates in other lanes. Since the maximum flow in the passing lane occurs in uncongested flow, the typical flow-density (or flow-occupancy) relationship for the inner lane has a reverse-lambda shape. Flow relationships for other lanes have triangular or inverted-V shapes.
3. The decrease in flow in the median lane is related to an increase in headway that occurs when the speed segregation breaks down. In Daganzo's interpretation, this is due to a "loss of motivation" by drivers in the fast lane because they can no longer pass other traffic. The concept of "loss of motivation" is unique to Daganzo's theory and may be regarded as its most important distinguishing feature.
4. Wave speeds in congested flow (i.e., between two flow states, both of which are fully congested) are constant and identical for all lanes. In particular, such wave speeds do not vary with flow or vehicular speed. "Fast waves" with velocities greater than this characteristic wave speed are believed to be possible, but are held in Daganzo's theory to exist only in (or on the borders of) semi-congested states in which speed segregation has not completely broken down.

Findings of the literature review that are related to the bases of Daganzo's theory may be summarized as follows:

Traffic in uncongested flow is segregated by lane in terms of speed, with the highest speeds typically found in the median lane. This speed segregation breaks down in the transition to congested flow. As a general rule, past empirical research supports this view (4,5), although the issue does not appear to have been studied extensively. The primary source of evidence is time series of speed. Where these appear in the literature, they almost always display this feature. The possible exception is that there are some reports of congested traffic which do not display speed synchronization. This occurs, for instance in "wide jams" as reported by Kerner and Rehborn (4) and in one case of "synchronized" flow reported by Kerner (6). In this latter case, however, there may be some question

about the overall circumstances and whether or not flow is “congested” in the sense adopted here.

The maximum flow rate in the passing lane (typically the lane nearest the freeway median) occurs in prior to the transition to congested flow and is greater than the maximum flow rates in other lanes. Since the maximum flow in the passing lane occurs in uncongested flow, the typical flow-density (or flow-occupancy) relationship for the inner lane has a reverse-lambda shape. Flow relationships for other lanes have triangular or inverted-V shapes. Evidence related to this proposition includes published plots of flow-concentration data (5,7-18) and a series of studies related to transitions from uncongested to congested flow at bottlenecks (13, 19-26). It is most often true that the maximum flow rate at a given site occurs in the inner lane in uncongested flow. All studies of transitions to congested flow that document the time series of flow on a lane-by-lane basis report a decrease in flow in the most heavily loaded lane, although this is not always the inner lane (23). It is not universally true, however, that the inner-lane flow-concentration relationship has a reversed-lambda shape and that those in other lanes have inverted-V shapes. In flow-concentration relationships reported in the literature to date, reversed-lambda and inverted-V shapes occur with about equal frequency in the median lane (5,9-11, 16-18). Relationships for the shoulder lane are sometimes inverted-V (5,14), but inverted-U shapes are also encountered (15,16). Also, reversed-lambda flow-concentration relationships do not necessarily imply that maximum sustainable uncongested flow exceeds the (theoretical) maximum congested flow (that is, that which results from extending a line representing the central tendency of the congested flow data until it intersects the uncongested flow line). Rather, there is evidence that the variance of flow decreases in the transition to congested flow, so that at least some of the “stub” of the lambda pattern may be the result of greater scatter in the uncongested-flow data (21,23). It should be emphasized that none of this necessarily invalidates Daganzo’s theory, but it does indicate that it may be of limited applicability because some of the conditions it assumes are not always present.

The decrease in flow in the median lane is related to an increase in headway that occurs when the speed segregation breaks down. In Daganzo’s interpretation, this is due to a “loss of motivation” by drivers in the fast lane because they can no longer pass other traffic. The concept of “loss of motivation” is unique to Daganzo’s theory and may be regarded as its most important distinguishing feature. Evidence related to the key concept of loss of motivation is mixed. Although it is generally true that headways increase in the most heavily loaded lane (the evidence is actually that flows decrease, but this implies an increase in headway), there is no clear evidence that this results from driver behavior in response to speed synchronization. If the change in headway were due to driver behavior (rather than the increase in passage time that accompanies the rather sharp speed drop that occurs in the transition to congested flow – see the discussion related to Hypothesis 8 in the preceding section) average time gaps should increase in the transition, and remain greater in congested flow than in uncongested flow. Banks (27) found that they did not. On the other hand, Dijkster et al (9) found that distance gaps were greater in congested flow than in uncongested flow, and since speeds are less, time gaps must also have increased in the transition. Also, data developed as part of this project

show that time gaps sometimes increase and sometimes decrease (or remain the same) in the transition to congested flow (28). As a result, the best that can be said is that in some cases, the evidence is consistent with the idea of a “loss of motivation” but in others it is not.

Wave speeds in congested flow (i.e., between two flow states, both of which are fully congested) are constant and identical for all lanes. In particular, such wave speeds do not vary with flow or vehicular speed. “Fast waves” with velocities greater than this characteristic wave speed are believed to be possible, but are held in Daganzo’s theory to exist only in (or on the borders of) semi-congested states in which speed segregation has not completely broken down. In this case, the preponderance of the evidence is against the idea that wave speeds are literally constant and that there is a characteristic wave speed that applies universally to congested flow. Daganzo’s theory does not require this, however, but only suggests that congested-flow wave speeds do not vary with the flow level, and that they are similar in all lanes for a given roadway segment. Some recent German work maintains that there is a characteristic speed, either for all types of waves (29) or for “wide jams” (4,17). In the latter case, however, it is also stated that this speed varies by roadway segment, and with weather conditions (30). Evidence from Mika (31), Koshi (5), Iwasaki (14), Kerner (32), and Forbes and Simpson (33) document considerable variation. The prevalence of speed synchronization in congested flow would seem to indicate that wave speeds in different lanes are nearly identical in a given disturbance (that is, waves tend to involve the whole roadway rather than individual lanes), but the evidence in Forbes and Simpson, in particular, suggests that wave speeds are widely variable over short distances, and that the speed of an individual wave can change over time.

The review of empirical research related to traffic flow also demonstrated that Daganzo’s theory, as presently developed, is of very limited scope when compared with the full range of phenomena considered in the literature. In some cases, this is probably deliberate and may even be an advantage, since its simplicity is one of its attractive features. In other cases, there may be potential for extending the theory to explain a wider range of phenomena, but so far this has not been done. One practical effect of this narrow scope is that it may be difficult to find sites and data sets that fully match the assumptions of the theory.

Phenomena described in the empirical literature that might possibly be addressed by extensions to the theory include the following:

1. The possible influence of ramps on lane use behavior, which is the subject of research by Hess (34, 35) and Roess and Ulerio (36). Daganzo provides a theory of merging and lane changing behavior by rabbits at freeway entrances, but does not discuss their exiting behavior. Lack of a theory of exiting behavior and how it affects the distribution of different driver classes in particular lanes limits the ability to test the theory, since most potential study sites are fairly close to off-ramps.

2. Functioning of bottlenecks other than merges in which ramp traffic is varying. A beginning has been made in extending the theory to cover bottlenecks on metered freeways for which ramp flow is held constant but freeway flows are increasing. Several other types of bottlenecks are documented in the literature, however (21,27,37,38). Lack of theory about the functioning of a full range of bottleneck types limits ability to test the theory.
3. Evidence that the fraction of traffic in various lanes under uncongested conditions varies with traffic volume (39). As matters now stand, this is consistent with the behavioral assumptions of Daganzo only if the proportion of “rabbits” to “slugs” also varies with volume.
4. The dependence of flow characteristics, particularly speeds, on vehicle type as well as driver type (40). Daganzo’s theory of driver behavior makes no allowances for the influence of vehicle type on characteristics such as speed and acceleration. Consequently, there is no distinction between the behavior of an aggressive truck driver (whose vehicle is capable of only limited speed and acceleration) and a timid driver in a high-powered automobile – both are classified as “slugs.” It is certainly conceivable that there are situations in which they would behave differently.

Phenomena that are generally (and probably deliberately) beyond the scope of the theory include:

1. Various microscopic and mesoscopic traffic characteristics (41-53). Ignoring such phenomena restricts the theory to driver behavior averaged over a suitable number of drivers. This is an advantage in that predictions can be made without use of microscopic input data, but it may also lead to distorted concepts of driver behavior.
2. Evidence related to various characteristics of congested flow (5,17,29,33). As is typical of kinematic wave theories, Daganzo’s theory ignores the details of flow phenomena within queues.
3. Evidence related to the effects of location, weather, construction, incidents, etc. on flow characteristics (5,12,18,54-59). The major features of Daganzo’s theory can be adjusted to take these effects into account, but the theory provides no explanation for why these variations occur.

CASE STUDY I: SOUTHBOUND I-5 DOWNSTREAM OF MANCHESTER AVENUE, SAN DIEGO COUNTY

The study site is southbound I-5 from Manchester Avenue to Via de la Valle in San Diego County, California. A morning peak period bottleneck is located in this vicinity. The freeway configuration in this section is four southbound lanes; a fifth lane is added just south of Via de la Valle. There are on- and off-ramps at three interchanges, Manchester Avenue, Lomas Santa Fe Drive, and Via de la Valle. All on-ramps are metered. Ramp meter detectors (the source of the data) are located just upstream from the

on-ramps at the three interchanges. The Lomas Santa Fe detectors are located 1.35 mi (2.17 km) downstream from those at Manchester Avenue. Those at Via de la Valle are 1.13 mi (1.82 km) downstream from those at Lomas Santa Fe Drive. There is a significant upgrade between Manchester Avenue and Lomas Santa Fe Drive and a significant downgrade between Lomas Santa Fe Drive and Via de la Valle. Available data include traffic volumes and occupancies for individual freeway lanes, and ramp passage counts for individual ramp lanes, reported every 30 seconds. Data were analyzed in detail for 10 days in April and May 2001. One of these days was subsequently eliminated from consideration because an apparent incident upstream of the study site led to anomalous traffic conditions.

On all days and at all three locations, flow decreases were noted in the left lane at about the time of flow breakdown. Flow decreases were identified by means of re-scaled cumulative plots of flow. In almost all cases, these were characterized by a period in which the slope of the cumulative plot was increasing, followed by a period of 10 min or more during which the slope was virtually constant, followed by a distinct downward break in the slope, indicating a decrease in flow. The flow decreases were quantified by means of event-based averaging: time periods with near constant average flow before and after the flow decrease (around 10 minutes in most cases) were identified, and flows were averaged for these periods. Flow decreases were significant in all cases, amounting to an average of 17.7 percent at Via de la Valle, 23.8 percent at Lomas Santa Fe Drive, and 37.1 percent at Manchester Avenue.

Similar comparisons were made for individual lanes and for flow averaged across all lanes, using the same time periods as for the left lane. Flow averaged across all lanes also decreased when flow in the left lane decreased, but by a lesser amount. Flow across all lanes decreased on all days at Manchester Avenue. At Via de la Valle and Lomas Santa Fe Drive, the average flow across all lanes declined on 6 of 9 days. The flow increases at these two locations occurred on the same day only once, however. Comparison flow changes in each lane established that the flow decreases were almost always accompanied by a redistribution of flow across the lanes in which flow in at least one lane increased. This implies that the changes in the relative flow in the different lanes involved some traffic switching out of the more heavily traveled lanes, rather than merely a reduction in flow in these lanes.

Daganzo's theory assumes that the initiation of semi-congested states and (eventually) flow breakdown takes place downstream of the critical on-ramp. All the detector sites are upstream of on-ramps and hence do not reflect conditions at the presumed critical point. They also do not reflect the maximum flows across all lanes. Flows immediately downstream of the ramps were calculated by adding the on-ramp flow to the freeway flow measured at the detectors just upstream. These calculations were possible for only 7 days because some ramp counts were not available. The maximum flow per lane just prior to breakdown occurs downstream of the Manchester Avenue on-ramp. Daily flows in this section ranged from 2,307 veh/h to 2,438 veh/h, with an average of 2,370 veh/h, and always decreased immediately after the left lane flow drop at the detectors. Just

downstream of the Lomas Santa Fe on-ramp, pre-breakdown flows ranged from 2,074 veh/h to 2,397 veh/h, and flow decreased in only 3 cases out of 7.

In most cases, the left-lane flow decreases coincided roughly with decreases in speed and increases in average time gaps in this lane. In some cases, they also coincided (again, roughly) with synchronization of speeds across the lanes. At Via de la Valle, data were about evenly split between cases in which the flow drop preceded the initial speed drop (5 cases) and those in which the speed drop was first (4 cases). At Lomas Santa Fe Drive, the speed drop tended to precede the flow decrease (4 cases out of 9) or to coincide with it (4 cases). At Manchester Drive, the speed decrease tended to precede the flow decrease (8 of 9 cases).

Speed synchronization tended to take place after the initial flow decrease. In several cases it was brief, existing during the slowest parts of the speed disturbance but not during the subsequent recovery. This was true even when speed recovered to a level considerably less than the uncongested speed. To some extent this tendency may be the result of the nature of the data: all speeds are estimated from volumes and occupancies, and these estimates are subject to biases where large vehicles are present. Most of the large vehicles are in lanes 3 and 4, and these have significantly lower estimated speeds than lanes 1 and 2 except in the slowest parts of the speed oscillations. It is not clear how much of the speed differential is real and how much is due to the biases in the estimates.

Because of the large distances between the detectors, it was difficult to determine the exactly where the flow and speed disturbances began. Two types of information were used to try to pinpoint the approximate point of origin of the flow breakdown. First, the times that the left-lane flow decreases were recorded at the different detector stations were compared. In theory, events of this nature should be transmitted downstream at approximately the speed of traffic and upstream at some smaller wave speed. Consequently, the first detector at which the flow decrease was noted should be the closest one to the origin of the disturbance. If this method is used, however, it is not possible to be completely certain whether the disturbance occurred upstream or downstream from the closest detector. A second test was to compare re-scaled cumulative plots of flow and occupancy. For detectors upstream of the disturbance, the expected pattern is for the decrease in flow in the left lane to correspond to an increase in occupancy in the same lane; for detectors sufficiently far downstream of the disturbance, increases and decreases in flows and occupancies should coincide with one another. For locations in the zone of acceleration immediately downstream of the origin of the disturbance, however, this test is not reliable since there might still be an increase in occupancy coinciding with the decrease in flow.

Both methods sometimes yielded unexpected results. For instance, the time lags sometimes indicated that either the disturbances were moving downstream at much lower speeds than expected, or else that left-lane flow decreases at successive detectors were independent events. Close inspection of the re-scaled cumulative flow and occupancy plots resulted in a variety of intermediate patterns as well as the extremes described above – for instance, occupancy increases sometimes lagged flow decreases by a minute

or two and at other times led them; also, occupancy sometimes decreased with flow for a short time and then increased. Results of “best guess” estimates of the location of the initial disturbance using the two methods agreed about two-thirds of the time. Based on these, it appears that the disturbances originated in different sections on different days. In all cases, they appeared to occur downstream from the Manchester Avenue detectors; in roughly half the cases, the most probable location was between Manchester Avenue and Lomas Santa Fe Drive (where the flow per lane was greatest), but in other cases the probable point of breakdown was either between Lomas Santa Fe Drive and Via de la Valle or even downstream of Via de la Valle.

Hypotheses that could be tested at this location included Hypotheses 6, 8, 10, 11, and 12. Hypothesis 7 could not be tested because none of the detector stations was immediately downstream from a potentially critical ramp. Hypothesis 9 was testable only if (or when) the Manchester on-ramp was critical; although all the on-ramps in the vicinity are metered, only at Manchester Avenue was the ramp flow great enough for the meter to stabilize it prior to the initial flow breakdown (in other words, flows on the other ramps were less than the minimum metering rate and still increasing at time of flow breakdown).

Results are as follows:

6. *An observer a considerable distance downstream of a merge bottleneck will see first a drop in flow in the passing lanes but with speed in the passing lanes greater than in the shoulder lane, then a reduction of flow in the passing lane, a more even distribution of flow across the lanes, and an increase in overall flow.*

Although it is not altogether clear which detector station was at the downstream end of the critical section, the entire sequence predicted by this hypothesis was not observed at any location. In particular, there was no preliminary decrease in flow in the passing lanes indicating the existence of a semi-congested state. Rather, the initial decreases in flow in the left lane were accompanied in almost every case by the redistribution of flow across the lanes that was expected to result from the subsequent flow collapse. Moreover, speed synchronization, although it did sometimes occur, was not closely associated with any of the other events in the sequence, except at Manchester Avenue, which was clearly upstream of the point of flow breakdown. Finally, the redistribution of flow did not consistently contribute to any increase in the overall flow. Rather, at the locations that appeared to be downstream of the point of flow breakdown (Via de la Valle and Lomas Santa Fe Drive), there was an overall tendency for flow to decrease despite the redistribution, although this did not occur every day.

8. *Average time gaps in the passing lane will increase when speeds in the passing lane drop to the level of those in other lanes.*

Increases in time gaps were consistently observed at about the time of the left-lane flow decreases and the redistribution of flow across the lanes. For the most part, however, they occurred *before* speed synchronization, which might indicate that speed synchronization

(when it occurred) was the result of the other changes in speed, flow, and vehicle spacing, rather than the cause, as implied by Daganzo's theory.

9. *If meters hold the on-ramp flows steady before the time of flow breakdown, the actual flow collapse will take place upstream of the critical ramp. Semi-congested states (involving a small drop in flow and speed in the left lane, but speeds in the left lane higher than those in the right lane) will be observed at detector locations upstream of the critical ramp prior to the flow collapse.*

This hypothesis was not confirmed. It is not clear whether it could have been tested at this location or not. The only location that met the condition that the meters hold the ramp flow steady prior to flow breakdown was Manchester Avenue. Clearly, the flow breakdown was downstream of this location, which would contradict the hypothesis. On the other hand, it is not clear that the Manchester Avenue on-ramp was (or was always) the critical ramp. The evidence suggests that there was no real consistency in the location of flow breakdown but that it sometimes occurred downstream of the Lomas Santa Fe Drive on-ramp and sometimes downstream of the Via de la Valle on-ramp. When this was the case, the hypothesis was untestable.

Scatter plots of flows and occupancies were prepared for individual lanes at each site for each day for which data were available. These provided tests for Hypotheses 10 through 12.

10. *Flow-concentration plots for the left lane at locations downstream from bottlenecks will display a reversed-lambda shape.*

This hypothesis was confirmed, although the reversed-lambda shape was not *always* observed in the left lane at these locations. Reversed-lambda shapes were observed in 6 of 10 cases at Via de la Valle, and 9 of 10 cases at both Lomas Santa Fe Drive and Manchester Avenue. In the other cases, the left-lane pattern was inverted V.

11. *Flow-concentration plots for the right lane will display an inverted-V shape; the central tendency of the congested-regime data will be parallel to that of the left lane, and will be displaced downward (i.e., lower flow for the same occupancy for the entire congested flow range).*

This hypothesis was generally not supported. At Via de la Valle there was insufficient congested-flow data to identify any patterns. Also, at this location, lane usage is somewhat unusual. A fifth lane is added immediately downstream, and flow in lane 4 (numbering outward from the freeway median) is greater than flow in lane 3. At Lomas Santa Fe Drive, there were also several days for which the pattern could not be identified. Of the 6 cases that could be identified, 4 displayed an inverted-V shape and 2 an inverted U. At Manchester Drive, the inverted-U shape was found in 9 of 10 cases, and the inverted V in one case. There was no clear indication of downward displacement in the right lane data. At Manchester Avenue, where there is abundant congested flow data, the outer envelopes of the congested flow data appear to be similar for all lanes; however,

there is more scatter towards the inside of the plot (i.e., lower occupancy for identical flow) for the outer lanes. On the other hand, comparisons of average time gaps in the different lanes suggest that there may be some downward displacement in the central tendency of the congested flow-occupancy relationship for the outer lanes and that this is most pronounced in the right lane.

12. *Flow-concentration plots for intermediate lanes may be either reversed-lambda or inverted-V. Their shapes and displacements will indicate whether they are to be considered passing lanes, slow lanes, or some intermediate type with hypothetical driver types of their own.*

This hypothesis was generally confirmed. The predominant pattern for the intermediate lanes at all locations was inverted-V. All patterns that could be identified were inverted-V except for one case in which the pattern for lane 2 was reversed-lambda.

Summary

The hypotheses related to the flow-occupancy diagrams are only partly supported. The hypothesis that the flow-occupancy plot for the left lane is reversed-lambda is supported, although not universally. The hypothesis that the flow-occupancy plot for the right lane is inverted-V is generally *not* supported. The flow-occupancy plots for lanes 2 and 3 are predominantly inverted-V. The hypothesis that the congested flow portion of the right lane flow-occupancy plot is displaced downward relative to that of the left lane is not *clearly* supported; also, it is not clear whether this effect occurs for lanes 2 and 3.

Of the hypotheses related to the sequence of events surrounding flow breakdown, Hypothesis 6 was not supported, Hypothesis 7 was not testable, Hypothesis 8 was partly supported, and Hypothesis 9 was either not supported or not testable.

CASE STUDY II: WESTBOUND I-8 DOWNSTREAM OF FLETCHER PARKWAY, SAN DIEGO AND LA MESA

The study site is westbound I-8 from Fletcher Parkway to College Avenue in the cities of San Diego and La Mesa, California. A morning peak period bottleneck is located in this vicinity. The freeway configuration in this section is four westbound lanes between the Fletcher Parkway and Lake Murray Boulevard-70th Street interchanges; a fifth lane is added just west of the 70th Street on-ramp and continues past College Avenue. There are on- and off-ramps at three interchanges, Fletcher Parkway, Lake Murray Boulevard-70th Street, and College Avenue. All on-ramps are metered. Ramp meter detectors (the source of the data) are located just upstream from the on-ramps at the three interchanges. The Lake Murray Boulevard-70th Street detectors are located 0.88 mi (1.42 km) downstream from those at Fletcher Parkway. Those at College Avenue are 1.36 mi (2.20 km) downstream from those at Lomas Santa Fe Drive. Grades throughout the study site are negligible, except that the College Avenue detectors are at the beginning of a downgrade of approximately 5 percent. Available data include traffic volumes and occupancies for

individual freeway lanes, and ramp passage counts for individual ramp lanes, reported every 30 seconds. Data were analyzed in detail for 9 days in October 2001.

On most days, flow decreases were noted in the left lane at about the time of flow breakdown at all three locations. Left-lane flow decreases were not observed at College Avenue on October 15 or October 19 or at Lake Murray Boulevard-70th Street on October 19. Flow decreases were identified by means of re-scaled cumulative plots of flow. In almost all cases, these were characterized by a period in which the slope of the cumulative plot was increasing, followed by a period of 10 min or more during which the slope was virtually constant, followed by a distinct downward break in the slope, indicating a decrease in flow. The flow decreases were quantified by means of event-based averaging: time periods with near constant average flow before and after the flow decrease (around 10 minutes in most cases) were identified, and flows were averaged for these periods. Table 1 compares the average flows in the left lane immediately before and immediately after the flow decreases. These are substantial in all cases although not as great as in the I-5 case study, amounting to an average of 10.1 percent at College Avenue, 12.4 percent at Lake Murray Boulevard-70th Street, and 15.2 percent at Fletcher Parkway.

Similar comparisons were made for individual lanes and for flow averaged across all lanes, using the same time periods as for the left lane. Flow averaged across all lanes also decreased when flow in the left lane decreased, but by a lesser amount. Flow across all lanes decreased on all but one day at Fletcher Parkway. At College Avenue, the average flow across all lanes declined on 4 of 7 days; at Lake Murray Boulevard-70th Street it decreased on 7 of 8 days. Flow increases occurred at all three locations on October 18, and on additional days at College Avenue. Comparison flow changes in each lane established that the flow decreases were almost always accompanied by a redistribution of flow across the lanes in which flow in at least one lane increased. This implies that the changes in the relative flow in the different lanes involved some traffic switching out of the more heavily traveled lanes, rather than merely a reduction in flow in these lanes.

Daganzo's theory assumes that the initiation of semi-congested states and (eventually) flow breakdown takes place downstream of the critical on-ramp. All the detector sites are upstream of on-ramps and hence do not reflect conditions at the presumed critical point. They also do not reflect the maximum flows across all lanes. Flows immediately downstream of the ramps were calculated by adding the on-ramp flow to the freeway flow measured at the detectors just upstream. These calculations were possible for only 7 days because some ramp counts were not available. The maximum flow per lane just prior to breakdown occurs downstream of the Fletcher Parkway on-ramp. Daily flows in this section ranged from 2,023 veh/h to 2,365 veh/h, with an average of 2,253 veh/h, and decreased immediately after the left lane flow drop at the detectors in all but one case. Just downstream of the Lake Murray Boulevard-70th Street on-ramp, pre-breakdown flows ranged from 1,906 veh/h to 2,159 veh/h, (across 5 lanes as compared with 4 lanes downstream of Fletcher Parkway) and flow decreased in 5 cases out of 9.

As in the I-5 case study, the flow decreases usually coincided roughly with speed decreases in the left lane; however, left-lane time gap increases were *not* observed at any of these locations. At College Avenue and Lake Murray Boulevard-70th Street, the initial speed drop tended to precede the flow drop. At Fletcher Parkway, the data are about evenly divided between cases in which the speed drops lead and those in which they lag the flow drop and all the lags are relatively small.

In some cases, the left lane flow drops also coincided (again, roughly) with synchronization of speeds across the lanes. At Lake Murray Boulevard-70th Street, speed synchronization tended to take place before the initial left-lane flow decrease, but at Fletcher Parkway, it tended to take place after the initial flow decrease. At College Avenue, speed synchronization was normally not observed. This may be due, in part, to anomalies in the estimated speeds at College Avenue, since these did not display the usual speed differentials in which the highest speeds are in the left lane. In the one case where speed synchronization was observed, it appears to have been due to an event that was not related to the initial left lane flow decrease.

As in the case of the I-5 case study, speed synchronization tended to be brief, existing during the slowest parts of the speed disturbance but not during the subsequent recovery. As in that case it is not clear how much of the speed differential is real and how much is due to the biases in the estimates.

Because of the large distances between the detectors, it was difficult to determine the exactly where the flow and speed disturbances began. Two types of information were used to try to pinpoint the approximate point of origin of the flow breakdown. First, the times that the left-lane flow decreases were recorded at the different detector stations were compared. In theory, events of this nature should be transmitted downstream at approximately the speed of traffic and upstream at some smaller wave speed. Consequently, the first detector at which the flow decrease was noted should be the closest one to the origin of the disturbance. If this method is used, however, it is not possible to be completely certain whether the disturbance occurred upstream or downstream from the closest detector. A second test was to compare re-scaled cumulative plots of flow and occupancy. For detectors upstream of the disturbance, the expected pattern is for the decrease in flow in the left lane to correspond to an increase in occupancy in the same lane; for detectors sufficiently far downstream of the disturbance, increases and decreases in flows and occupancies should coincide with one another. For locations in the zone of acceleration immediately downstream of the origin of the disturbance, however, this test is not reliable since there might still be an increase in occupancy coinciding with the decrease in flow.

In this case study, flow decreases were normally observed first at College Avenue; however, these appeared in most cases to be independent of those farther upstream. Time lags between College Avenue and Lake Murray Boulevard were typically smaller than would be expected with any reasonable upstream wave speed. Moreover, only two of the cases at College Avenue displayed a pattern in which occupancy increased when flow decreased, indicating that either no real flow breakdown was associated with the flow

decreases, or else that the location was downstream of the flow breakdown. Since there was no real evidence of flow breakdown upstream that coincided with these flow drops, they may have been just random fluctuations. Comparisons of time lags and patterns of variation in flow and occupancy at Lake Murray Boulevard-70th Street and Fletcher Parkway suggested that the origin of the disturbances in that vicinity was close to the Lake Murray Boulevard-70th street detectors, but it was not entirely clear whether it was upstream or downstream. In most cases, however, the time lags are consistent with the hypothesis that the disturbances originated between Fletcher Parkway and Lake Murray Boulevard-70th Street.

Hypotheses that could be tested at this location included Hypotheses 6, 8, 10, 11, and 12. Hypothesis 7 could not be tested because none of the detector stations was immediately downstream from a potentially critical ramp. It is not clear whether Hypothesis 9 was really testable because it is not altogether clear whether the meters were holding ramp flows constant immediately before flow breakdown or not.

Results are as follows:

6. *An observer a considerable distance downstream of a merge bottleneck will see first a drop in flow in the passing lanes but with speed in the passing lanes greater than in the shoulder lane, then a reduction of flow in the passing lane, a more even distribution of flow across the lanes, and an increase in overall flow.*

It is not altogether clear which detector station was at the downstream end of the critical section, however, as was the case in the I-5 case study, the entire sequence predicted by this hypothesis was not observed at any location. In particular, there was no preliminary decrease in flow in the passing lanes indicating the existence of a semi-congested state. Rather, the initial decreases in flow in the left lane were accompanied in almost every case by the redistribution of flow across the lanes that was expected to result from the subsequent flow collapse. Moreover, speed synchronization, although it did sometimes occur, was not closely associated with any of the other events in the sequence, except at Fletcher Parkway, which was clearly upstream of the point of flow breakdown. In this case, however, speed synchronization at the most likely critical location (Lake Murray Boulevard-70th Street) did precede the flow decrease in most cases. Finally, the redistribution of flow did not consistently contribute to any increase in the overall flow. Rather, at all locations, there was an overall tendency for flow to decrease despite the redistribution, although this did not occur every day.

8. *Average time gaps in the passing lane will increase when speeds in the passing lane drop to the level of those in other lanes.*

This hypothesis was not confirmed at this location. No increases in average time gaps in the left lane were observed to occur at the times of the initial left-lane flow decreases. It should be noted that if the flow-occupancy diagrams from I-8 are compared with those from I-5, the average time gaps in congested flow on I-8 tend to be smaller than those on I-5, although it is not clear why this should be the case.

9. *If meters hold the on-ramp flows steady before the time of flow breakdown, the actual flow collapse will take place upstream of the critical ramp. Semi-congested states (involving a small drop in flow and speed in the left lane, but speeds in the left lane higher than those in the right lane) will be observed at detector locations upstream of the critical ramp prior to the flow collapse.*

This hypothesis was not confirmed. It is not clear whether it could have been tested at this location or not. Although ramp flow appeared to stabilize about the time of flow breakdown or a little before at all three locations, it subsequently decreased (apparently due to a lack of demand, and ramp flow patterns to not clearly establish that the meters were holding the ramp flows steady. In any event, the critical ramp was most likely that at Fletcher Parkway, and flow breakdown clearly took place downstream from this location.

Scatter plots of flows and occupancies were prepared for individual lanes at each site for each day for which data were available. These provided tests for Hypotheses 10 through 12.

10. *Flow-concentration plots for the left lane at locations downstream from bottlenecks will display a reversed-lambda shape.*

This hypothesis was *not* confirmed. Flow-occupancy relationships for the left lane were classified as predominantly inverted-V at all three locations (7 of 9 at Fletcher Parkway, 8 of 9 at Lake Murray Boulevard-70th Street, and 5 of 6 at College Avenue). In the remaining cases where there was enough congested flow data to permit classification, reversed-lambda patterns were observed.

11. *Flow-concentration plots for the right lane will display an inverted-V shape; the central tendency of the congested-regime data will be parallel to that of the left lane, and will be displaced downward (i.e., lower flow for the same occupancy for the entire congested flow range).*

This hypothesis was supported at some locations but not others. Lane 4 patterns were always inverted-V at Lake Murray Boulevard-70th Street, but predominantly inverted-U at Fletcher Parkway (7 of 9, with 2 inverted-V). Patterns in lane 5 at College Avenue were predominantly inverted-V (3 of 4 with 1 inverted-U). As at Manchester Avenue in the I-5 case study, the outer envelopes of the congested flow data appear to be similar for all lanes at Fletcher Parkway, where there is abundant congested flow data; once again, however, there is more scatter towards the inside of the plot (i.e., lower occupancy for identical flow) for the outer lanes. Consequently, the hypothesis that the congested flow portion of the plot for the right lane is displaced downward is not *clearly* supported at this location.

12. *Flow-concentration plots for intermediate lanes may be either reversed-lambda or inverted-V. Their shapes and displacements will indicate whether they are to*

be considered passing lanes, slow lanes, or some intermediate type with hypothetical driver types of their own.

This hypothesis was not generally confirmed. The flow-occupancy plots for the intermediate lanes include all three patterns, but are most commonly inverted-V or inverted U.

Summary

The hypotheses related to the flow-occupancy diagrams are only partly supported. The hypothesis that the flow-occupancy plot for the left lane is reversed-lambda is not supported. The hypothesis that the flow-occupancy plot for the right lane is inverted-V is supported at some locations but not others. The flow-occupancy plots for intermediate lanes include all three types, but are most commonly inverted-V or inverted-U. The hypothesis that the congested flow portion of the right lane flow-occupancy plot is displaced downward relative to that of the left lane is not *clearly* supported; also, it is not clear whether this effect occurs for lanes 2 and 3.

Of the hypotheses related to the sequence of events surrounding flow breakdown, Hypothesis 6 was not supported, Hypothesis 7 was not testable, Hypothesis 8 was not supported, and Hypothesis 9 not supported.

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